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# EXTENSION TABLES FOR THE U.S. STANDARD ATMOSPHERE, 1962, WITH SPECIAL ATTENTION TO THE CALCULATION OF GEOPOTENTIAL

R. A. Minzner

GCA CORPORATION

Bedford, Massachusetts



SCIENTIFIC REPORT NO. 3

CONTRACT NO. AF19(628)-6085

PROJECT NO. 5710

Contract Monitor

Frank A. Marcos

Upper Atmosphere Physics Laboratory



*Prepared for*

AIR FORCE CAMBRIDGE RESEARCH LABORATORIES

OFFICE OF AEROSPACE RESEARCH

UNITED STATES AIR FORCE

BEDFORD, MASSACHUSETTS



January 1967

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Washington, D.C.

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# PREFACE

This report is submitted in fulfillment of items 2 and 3 of Contract AF19(528)-6085 monitored for AFCRL by Frank Marcos. It presents a set of tables consisting of an extension of the U.S. Standard Atmosphere in the altitude region of 90 to 120 km for which region tables had originally been published only as a function of integral multiples of one geometric kilometer. The tables in this report are presented as a function of integral multiples of one standard geopotential kilometer and includes a discussion of the various equations and constants involved.

This report also includes the development of a simplified function for accurately relating geopotential and geometric altitude.

## ABSTRACT

The "United States Standard Atmosphere, 1962", was published with two kinds of metric-unit tables for the altitude interval from -5000 to 90,000 meters. One kind of table presented the atmospheric properties as a function of integral multiples of particular numbers of geopotential meters while the second presented the atmospheric properties as a function of integral multiples of similar numbers of geometric meters. For the region above 90,000 meters, altitude only one type of metric table was published. This type presented atmospheric properties in integral multiples of particular numbers of geometric meters. A similar situation prevailed for the English-unit tables. The need for both metric-unit and English-unit tables as a function of integral multiples of specific numbers of geopotential meters for altitudes above 90 kilometers has prompted a new set of calculations, which required the use of equations not specifically presented in the United States Standard Atmosphere, 1962. The development of these equations is discussed and the value of all constants employed are given. The calculations involve a transformation between geopotential and geometric altitude, and the development of an empirical analytical expression relating these quantities is presented. This empirical function yields results which differ by less than 0.1 meter at 700 km altitude, from those computed in an unspecified manner for the United States Standard Atmosphere, 1962.

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## SECTION I

## PRESSURE AND DENSITY EQUATIONS FOR TEMPERATURE-ALTITUDE PROFILES LINEAR WITH RESPECT TO GEOMETRIC ALTITUDE

Above 90 km, the pressure-altitude profile of the U.S. Standard Atmosphere, 1962(Ref. 1) is defined in terms of a segmented temperature-altitude profile for which each segment is linear in terms of geometric altitude, i.e.,

$$T_M = T_{M_r} + L(Z-Z_r) \tag{1}$$

where

$T_M$ is the molecular scale temperature at altitude Z

$T_{M_r}$ is the reference molecular scale temperature at reference altitude $Z_r$

$Z_r$ is the base of any layer characterized by a single constant value of L

L is the gradient of $T_M$ with respect to Z, i.e., $dt_M/dZ$.

This condition is in contrast to that below 90 km where the segmented temperature-altitude profile has segments which are linear in terms of geopotential i.e.,

$$T_M = T_{M_r} + L' \, (H-H_r) \tag{2}$$

where

$T_M$ is the molecular scale temperature at geopotential H

$T_{M_r}$ is the molecular scale temperature at geopotential $H_r$

$H_r$ is the base of any layer characterized by a single constant value of L

L′ is the gradient of $T_M$ with respect to H, i.e., $dT_M/dH$.

Equation I.2.10-(3) of the document of the U.S. Standard Atmosphere, 1962 is suitable for calculating pressures associated with a segmented temperature-altitude profile having segments which are linear with respect to geopotential, but is not suitable for calculating Standard-Atmosphere pressures above 90 km where the temperature-altitude profile is defined to have segments which are linear with respect to geometric altitude. For the region above 90 km the Standard-Atmosphere document does not contain a detailed equation for calculating

pressure or density but presents only a general integral form, not suitable for direct numerical evaluation. The equation actually used for calculating the standard-atmosphere pressures for integral geometric altitudes was probably a corrected form of one of a pair of equations published with some format errors by Champion and Minzner in 1963(Ref. 2). A redevelopment of those pressure-altitude equations and a related density-altitude equation show the correct forms to be as follows:

$$\frac{-RL}{M_o} \ln \frac{p}{p_r} = \left[ \ln \frac{T_{M_r} + L(z-z_r)}{T_{M_r}} \right] \left[ \sum_{i=0}^{6} (-1)^i g_i a^i \right]$$

$$+ \sum_{j=0}^{5} \left[ \frac{z^{j+1} - z_r^{j+1}}{j+1} \right] \left[ \sum_{i=j+1}^{6} (-1)^{i+j+1} g_i a^{i-j-1} \right] \qquad (3)$$

$$p = p_r \left[ \frac{T_{M_r} + L(z-z_r)}{T_{M_r}} \right]^{-\left[ \sum_{i=0}^{6} (-1)^i g_i a^i \right] \frac{M_o}{RL}}$$

$$\exp \left[ \frac{-M_o}{RL} \sum_{j=0}^{5} \left( \frac{z^{j+1} - z_r^{j+1}}{j+1} \right) \left( \sum_{i=j+1}^{6} (-1)^{i+j+1} g_i a^{i-j-1} \right) \right] \qquad ($$

$$\rho = \rho_r \left[ \frac{T_{M_r} + L(z-z_r)}{T_{M_r}} \right]^{-1-\left[ \sum_{i=0}^{6} (-1)^i g_i a^i \right] \frac{M_o}{RL}}$$

$$\exp \left[ \frac{-M_o}{RL} \sum_{j=0}^{5} \left( \frac{z^{j+1} - z_r^{j+1}}{j+1} \right) \left( \sum_{i=j+1}^{6} (-1)^{i+j+1} g_i a^{i-j-1} \right) \right] \qquad (5)$$

where

$R$ is the universal gas constant

$M_o$ is the sea-level value of the molecular weight

$p$ is the pressure at altitude $Z$

$p_r$ is the pressure at reference altitude $Z_r$

$\rho$ is the density at altitude $Z$

$\rho_r$ is the density at reference altitude $A_r$

$g_i$ is the coefficient of the ith term of Lambert's gravitational formula given below

$$g(Z) = \sum_{i=0}^{6} g_i Z^i$$

$$= 9.8066500 - 3.0854195 \times 10^{-6} Z + 7.2539455 \times 10^{-13} Z^2$$

$$-1.5167771 \times 10^{-19} Z^3 + 2.9724620 \times 10^{-26} Z^4$$

$$-5.5905936 \times 10^{-33} Z^5 + 1.0219762 \times 10^{-39} Z^6 \qquad (6)$$

At one point in the derivation of the above equations it was necessary to use the following interesting transformation in order to obtain a form suitable for integration with respect to Z:

$$\frac{K + AZ + BA^2 + CZ^3 + DZ^4 + EZ^5 + FZ^6}{a+Z} = \qquad (7)$$

$$\frac{1}{a+Z} [K - Aa + Ba^2 - Ca^3 + Da^4 - Ea^5 + Fa^6] +$$

$$Z [A - Ba + Ca^2 - Da^3 + Ea^4 - Fa^5] +$$

$$Z^2 [B - Ca + Da^2 - Ea^3 + Fa^4] +$$

$$Z^3 [C - Da + Ea^2 - Fa^3] +$$

$$Z^4 [D - Ea + Fa^2] +$$

$$Z^5 [E - Fa] +$$

$$Z^6 [F]$$

The pressure equation actually programmed in the digital machine calculation for tables presented in this report was Equation (4) of this document. The density was then computed from the gas law

$$\rho = \frac{P\,M_o}{T_M\,R} \tag{8}$$

using the values of millibar pressure computed from Equation (4) (multiplied by 100 to transform millibars to newtons per $m^2$) and the values of temperature computed from Equation (1).

## Constants and Boundary Conditions

The constants used in the calculation of the thermodynamic properties are those defined in the standard atmosphere:

$$R = 8.31432 \times 10^3 \text{ Joules } (^\circ K)^{-1} \text{ (kilomole)}^{-1}, \text{ and}$$

$$M_o = 28.9646 \text{ kg (kilomole)}^{-1},$$

such that

$$\frac{M_o}{R} = 3.483676 \times 10^3 \; ^\circ K \text{ sec}^2 \text{ m}^{-2}.$$

The values of $T_{M_r}$ and $p_r$ in the program are redefined for the base of each layer in accordance with the values tabulated in the Standard-Atmosphere publication.

| Geometric Altitude Meters | Temperature Degrees K | Temperature Gradient deg K/m | Pressure Millibars |
|---|---|---|---|
| 90,000 | 180.65 | 0.003 | $1.6438 \times 10^{-3}$ |
| 100,000 | 210.65 | 0.005 | $3.0075 \times 10^{-4}$ |
| 110,000 | 260.65 | 0.010 | $7.3544 \times 10^{-5}$ |
| 120,000 | 360.65 | 0.020 | $2.5217 \times 10^{-5}$ |

The coefficients of Lambert's equation for the acceleration of gravity are given in Equation (6).

The thermodynamic properties calculated in English units follow the defined conversions:

1 foot = 0.3048 meters,

1 pound = 0.45359237 kilogram,

1 Celcius degree = 1.8 Rankin degrees

a temperature of 0 degree Celsius = a temperature of 273.15°K.

From the above definitions the following derived relationships apply:

1 meter = 3.2808399 feet,

1 kg $m^{-3}$ = 6.242797 x $10^{-2}$ lbs $ft^{-3}$.

## SECTION II

## GEOPOTENTIAL TO GEOMETRIC ALTITUDE CONVERSION

The above-defined constants are sufficient to compute the Standard-Atmosphere values of pressure temperature and density as a function of geometric altitude between 90 and 150 geometric kilometers or between 295469.47 and 492125.98 geometric feet. The computation of these thermodynamic properties as a function of geopotential altitudes in a manner compatible with the relationships of the U.S. Standard-Atmosphere is a far more complicated problem, since the expressions used in this transformation in the standard atmosphere were never published in the Standard-Atmosphere document. In the absence of these expressions, a simple empirical correction term has been developed for use with the previously used simple expression for converting geopotential to geometric altitude. Thus, while the expression used by Minzner and Ripley in 1956 (Ref. 3) is

$$Z = \frac{r\,H}{r - H} \cdot \frac{g_o}{G} \qquad (9)$$

the revised expression is

$$Z = \frac{r\,[H + F(H)]}{r - [H + F(H)]} \cdot \frac{g_o}{G} \qquad (10)$$

where $F(H) = AA + ABH + ACH^2 + ADH^2 + ADH^3 + AEH^4$ (11)

where

$AA = 0.0000$

$AB = -0.2161710$

$AC = +0.1807561$

$AD = +0.9153012$

$AE = +0.2006785$

and where

$g_o/G$ is numerically but not dimensionally unity.

It is estimated that the differences between the values of Z for a given value of H as computed by Equation (11) and the values of Z for a given value of H as computed by the methods used in the Standard-Atmosphere do not exceed 0.2 meter over the entire altitude region of 0 to 700 kilometers. The development of Equations (10) and (11) and expressions satisfying the inverse of the relationship of Equation (10) is given in Appendix A.

## SECTION III

## CALCULATION OF THE TABLES

Equations (10) and (11) plus the previously discussed expressions, Equations (1), (4), (6), and (8) permit the computation of the desired extensions to the Standard-Atmosphere Tables, presented as Table 1 in metric units and as Table 2 in English units. Successive integral multiples of 1000 geopotential meters ranging from 90,000 to 120,000 are arbitrarily selected as values of H for the metric tables. These values are converted to the related values of Z through Equations (10) and (11). The resulting values of Z when introduced into Equation (4), into which the coefficients of Equation (6) have already been introduced, yield the values of pressure at altitude Z. The temperatures which are implicitly calculated in Equation (4) are independently calculated as a function of Z using Equation (1). The pressures and temperatures at a given value of Z then yield densities for that altitude through Equation (8).

In the case of the table presented in English units successive integral multiples of 5000 geopotential feet are selected, in the range 295,000 to 390,000, and are converted to geopotential meters. The procedure then follows that used for the metric tables except that the temperature, temperature gradient, and density are converted to the appropriate value in English units. The pressure is retained in millibars which is a metric-related unit. In addition, temperature is also calculated in degrees Celsius.

The above procedures have been specified in a fortran program presented in Appendix B.

TABLE 1

| GEOPT ALTITUDE | GEOMETRIC ALTITUDE | TEMP GRAD | MOLEC SCALE TEMP | PRESSURE | DENSITY |
|---|---|---|---|---|---|
| GEOPOTEN METERS | GEOMETRIC METERS | DEG K PER M | DEG K | MB | KG/CU M |
| 88743.3 | 90000.00 | .003 | 180.65 | .16438E-02 | .3170E-05 |
| 90000.00 | 91292.75 | .003 | 184.53 | .12993E-02 | .2453E-05 |
| 91000.00 | 92321.85 | .003 | 187.62 | .10814E-02 | .2007E-05 |
| 92000.00 | 93351.28 | .003 | 190.70 | .90272E-03 | .1649E-05 |
| 93000.00 | 94381.03 | .003 | 193.79 | .75574E-03 | .1358E-05 |
| 94000.00 | 95411.12 | .003 | 196.88 | .63448E-03 | .1122E-05 |
| 95000.00 | 96441.53 | .003 | 199.97 | .53413E-03 | .9304E-06 |
| 97000.00 | 98503.35 | .003 | 206.16 | .38151E-03 | .6446E-06 |
| 96000.00 | 97472.28 | .003 | 203.07 | .45083E-03 | .7734E-06 |
| 98000.00 | 99534.75 | .003 | 209.25 | .32364E-03 | .5388E-06 |
| 99000.00 | 100566.49 | .005 | 213.48 | .27529E-03 | .4492E-06 |
| 100000.00 | 101598.56 | .005 | 218.64 | .23502E-03 | .3744E-06 |
| 101000.00 | 102630.95 | .005 | 223.80 | .20139E-03 | .3134E-06 |
| 102000.00 | 103663.68 | .005 | 228.97 | .17318E-03 | .2634E-06 |
| 103000.00 | 104696.72 | .005 | 234.13 | .14942E-03 | .2223E-06 |
| 104000.00 | 105730.11 | .005 | 239.30 | .12934E-03 | .1882E-06 |
| 105000.00 | 106763.83 | .005 | 244.47 | .11230E-03 | .1600E-06 |
| 106000.00 | 107797.87 | .005 | 249.64 | .97802E-04 | .1364E-06 |
| 107000.00 | 108832.25 | .005 | 254.81 | .85413E-04 | .1167E-06 |
| 108000.00 | 109866.96 | .005 | 259.98 | .74796E-04 | .1002E-06 |
| 109000.00 | 110901.99 | .010 | 269.67 | .65732E-04 | .8491E-07 |
| 110000.00 | 111937.36 | .010 | 280.02 | .58048E-04 | .7221E-07 |
| 111000.00 | 112973.06 | .010 | 290.38 | .51494E-04 | .6177E-07 |
| 112000.00 | 114009.10 | .010 | 300.74 | .45873E-04 | .5313E-07 |
| 113000.00 | 115045.46 | .010 | 311.10 | .41025E-04 | .4593E-07 |
| 114000.00 | 116082.16 | .010 | 321.47 | .36824E-04 | .3990E-07 |
| 115000.00 | 117119.19 | .010 | 331.84 | .33167E-04 | .3481E-07 |
| 116000.00 | 118156.56 | .010 | 342.22 | .29970E-04 | .3050E-07 |
| 117000.00 | 119194.25 | .010 | 352.59 | .27163E-04 | .2683E-07 |
| 118000.00 | 120232.27 | .020 | 365.30 | .24691E-04 | .2394E-07 |
| 119000.00 | 121270.63 | .020 | 386.06 | .22544E-04 | .2034E-07 |
| 120000.00 | 122309.33 | .020 | 406.84 | .20682E-04 | .1771E-07 |

TABLE 2

| GEOPT ALTITUDE | GEOMETRIC ALTITUDE | TEMP GRAD | MOLEC SCALE TEMP | MOLEC SCALE TEMP | PRESSURE | DENSITY |
|---|---|---|---|---|---|---|
| GEOPOTEN FEET | GEOMETRIC FEET | DEG R PER FT | DEG R | DEG C | MB | LB/CU FT |
| 291152.56 | 295275.59 | .0016459 | 325.17 | -92.50 | .16438E-02 | [illegible] |
| 295000.00 | 299233.35 | .0016459 | 331.68 | -88.87 | .13197E-02 | [illegible] |
| 300000.00 | 304379.14 | .0016459 | 340.15 | -84.17 | .99843E-03 | [illegible] |
| 305000.00 | 309527.43 | .0016459 | 348.63 | -79.46 | .76096E-03 | [illegible] |
| 310000.00 | 314678.23 | .0016459 | 357.11 | -74.75 | .58319E-03 | [illegible] |
| 315000.00 | 319831.54 | .0016459 | 365.59 | -70.04 | .44992E-03 | .4817E-07 |
| 320000.00 | 324987.36 | .0016459 | 374.07 | -65.32 | .34920E-03 | .3654E-07 |
| 325000.00 | 330145.68 | .0027432 | 384.83 | -59.35 | .27266E-03 | .2773E-07 |
| 330000.00 | 335306.54 | .0027432 | 398.98 | -51.48 | .21466E-03 | [illegible] |
| 335000.00 | 340469.89 | .0027432 | 413.15 | -43.61 | .17041E-03 | .1634E-07 |
| 340000.00 | 345635.77 | .0027432 | 427.32 | -35.74 | .13635E-03 | .1249E-07 |
| 345000.00 | 350804.18 | .0027432 | 441.50 | -27.86 | .10988E-03 | .9743E-08 |
| 350000.00 | 355975.08 | .0027432 | 455.68 | -19.98 | .89169E-04 | .7660E-08 |
| 355000.00 | 361148.53 | .0154864 | 470.58 | -11.71 | .72821E-04 | .6057E-08 |
| 360000.00 | 366324.49 | .0154864 | 498.97 | 4.06 | .60017E-04 | [illegible] |
| 365000.00 | 371502.96 | .0154864 | 527.38 | 19.84 | .49997E-04 | .3711E-08 |
| 370000.00 | 376684.00 | .0154864 | 555.81 | 35.63 | .42051E-04 | .2961E-08 |
| 375000.00 | 381867.53 | .0154864 | 584.25 | 51.43 | .35674E-04 | .2390E-08 |
| 380000.00 | 387053.65 | .0154864 | 612.70 | 67.24 | .30502E-04 | .1940E-08 |
| 385000.00 | 392242.27 | .0154864 | 641.17 | 83.05 | .26267E-04 | [illegible] |
| 390000.00 | 397433.41 | .0109728 | 690.13 | 110.25 | .22802E-04 | .1293E-08 |

295275.59 GEOMETRIC FEET = 90000.00 GEOMETRIC METERS

## APPENDIX A

### DEVELOPMENT OF AN EMPIRICAL FUNCTION RELATING THE NUMERICAL VALUES OF GEOPOTENTIAL AND GEOMETRIC ALTITUDE AS PUBLISHED IN THE UNITED STATES STANDARD ATMOSPHERE

### A-1 COMPARISON OF DIFFERENT SETS OF CALCULATIONS OF GEOPOTENTIAL

Geopotential in the 1956 ARDC Model Atmosphere (Ref. A-1) was calculated by the expression

$$H_{56} = \frac{rZ}{r+Z} \cdot \frac{g_o}{G} \tag{A-1}$$

where $r = 6{,}356{,}766$ meters, and where $H_{56}$ is used to differentiate the results of this particular calculation of geopotential from the values tabulated in U.S. Standard Atmosphere, for which values the designation is $H_{62}$.

In the 1962 U.S. Standard Atmosphere (Ref. A-2), geopotential $H_{62}$ at any altitude Z was computed by the integration of a complicated gravity-acceleration expression along a path identical to the curved line of gravitational force passing through the point in question (where the point in question was defined relative to geometric latitude and relative to the distance from the center of an ellipsoid rather than relative to sea level) such that for each successive altitude, the integration must be performed along a different line of force. No specific equation suitable for direct numerical evaluation of $H_{62}$ as a function of Z was given in the standard-atmosphere document, nor has one been developed by the writer from the fundamental considerations which were given. Instead, a simple approximation formula in the nature of Equation (A-1) with a correction term was developed; i.e.,

$$H_{62/8} = \frac{rZ}{r+Z} \cdot \frac{g_o}{G} - f(Z) \tag{A-2}$$

where $H_{62/8}$ represents the eight-significant-figure values which when rounded lead to $H_{62/R}$, the values of geopotential published in the Standard Atmosphere.

A comparison of the tabulated six-significant-figure values (above 100,000 m) of $H_{62}$ with $H_{56/8}$ the eight-significant-figure values of $H_{56}$ as obtained from Equation (A-1) suggest the following observations:

(1) The tabulated values of $H_{62}$ are always less than the corresponding value of $H_{56/8}$ for altitudes greater than sea level.

(2) The tabulated values of $H_{62}$ are rounded from original seven- or eight-significant-figure values. (These rounded values will henceforth be designated as $H_{62/R}$).

(3) The values of $H_{62/8}$ which have been rounded to $H_{62/R}$ would most likely differ from $H_{56/8}$ in accordance with f(Z), some smooth monotonically increasing function of Z, which function has a value of zero at sea

level and a value of about 33 geopotential meters (m′) at the geometric altitude Z of 700 km; i.e.,

$$f(Z) = H_{56/8} - H_{62/8} \tag{A-3}$$

Thus, a curve fit to the difference between $H_{56/8}$ and $H_{62/8}$ (if these were available) would provide the desired function f(Z), and the desired approximation expression for calculating $H_{62/8}$ as in Equation (A-2) would have been determined. Unfortunately, values of $H_{62/8}$ are not available, and an indirect approach must be pursued.

## A-2 AN APPROACH FOR GENERATING f(Z)

An imaginary set of eight-significant-figure values of $H_{62}$ is hypothesized. If the hypothetical numerical values of $H_{62/8}$ were compared with Z in the region of sea level and immediately above, one would find that at Z = 0, $H_{62/8}$ = 0, and as Z increases above zero, $H_{62/8}$ would also increase but at a lesser rate than Z, so that at $Z = Z_{0.4999}$, which is the symbol for a specific altitude located somewhere between Z = 1750 and Z = 1800 meters, the value of $H_{62/8}$ would lag behind that of Z by exactly 0.4999 of a meter. At $Z = Z_{0.5000}$ which is the symbol for an altitude immediately above $Z_{0.4999}$, the value $(Z-H_{62/8})$ would become 0.5000. Between Z = 0 and Z = 4000 m, the value of $(Z-H_{62/8})$ would increase smoothly as Z increases in accordance with the values presented in Table A-1.

The exact numerical values of the symbolic altitudes $Z_{0.4999}$, $Z_{0.5000}$, $Z_{1.4999}$, etc. are not known, but from the U.S. Standard Atmosphere 1962 we may infer that these values are bounded within particular limits indicated in Table A-1. Thus $Z_{0.4999}$ and $Z_{0.5000}$ have values between 1750 and 1800 meters, while $Z_{1.4999}$ and $Z_{1.5000}$ would be found between the altitudes 3050 and 3100 meters, etc.

Returning momentarily to the reality of the U.S. Standard Atmosphere 1962, we can examine the difference between the tabulated integral values of geometric altitude $Z_i$ and the rounded values of geopotential altitude $H_{62/R}$ where $Z_i$ is increased discontinuously in integral steps of one meter as Z increases, with successive discontinuities occurring between $Z_{0.4999}$ and $Z_{0.5000}$, and again between $Z_{1.4999}$ and $Z_{1.5000}$, etc. as indicated in Table A-1. The differences $(Z-H_{62/8})$ are also tabulated.

Two differences given in Table A-1 are shown in Figure A-1 where the hypothetical quantity $(Z - H_{62/8})$ is shown as the solid-line, smooth-curve function, and the realistic quantity $(Z_i - H_{62/R})$ is shown as the discontinuous function represented as a series of alternate horizontal and vertical line segments, where these line segments connect the series of discrete points derived from the finite number of tabulated values. A graph of the difference $(Z - H_{56/8})$ is also shown as a smooth, dashed-line curve in Figure A-1 where $H_{56/8}$ represents the eight-significant-figure values obtained from Equation A-1.

TABLE A-1

DIFFERENCES $(A - H_{62/8})$ and $(Z_i - H_{62/R})$ AS A FUNCTION OF ALTITUDE

| Numerical Altitude (meters) | Symbolic Altitude | Difference $(Z - H_{62/8})$ (meters) | Difference $(Z_i - H_{62/R})$ (meters) |
|---|---|---|---|
| 0 | $Z_0$ | 0.0000 | 0 |
| 1750 | | | 0 |
| | $Z_{0.4999}$ | 0.4999 | 0 |
| | $Z_{0.5000}$ | 0.5000 | 1 |
| 1800 | | | 1 |
| 3050 | | | 1 |
| | $Z_{1.4999}$ | 1.4999 | 1 |
| | $Z_{1.5000}$ | 1.5000 | 2 |
| 3100 | | | 2 |
| 3950 | | | 2 |
| | $Z_{2.4999}$ | 2.4999 | 2 |
| | $Z_{2.5000}$ | 2.5000 | 3 |
| 4000 | | | 3 |

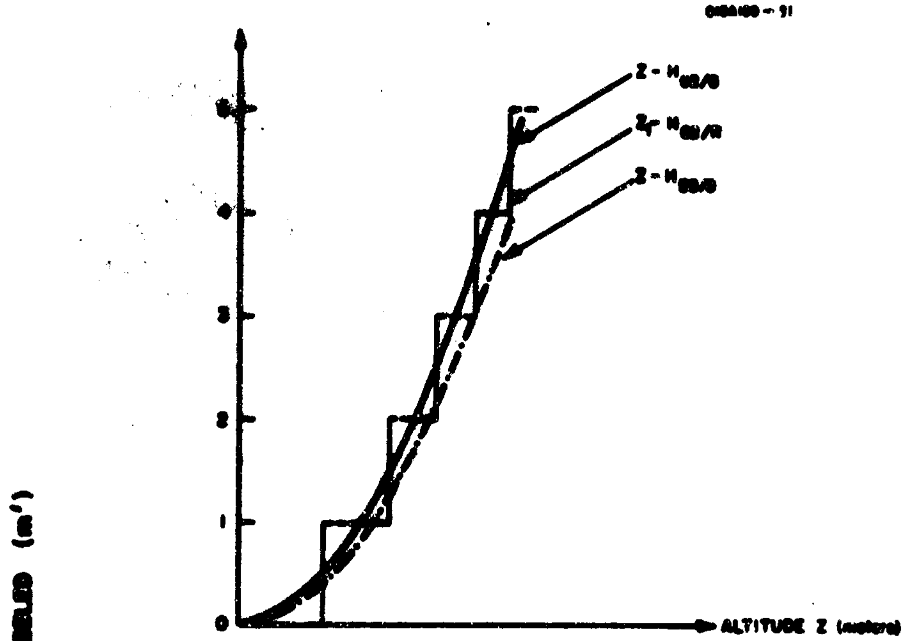


**Figure A-1** Differences $Z - H_{62/8}$, $Z_1 - H_{62/R}$ and $Z - H_{36/8}$ vs altitude.


**Figure A-2** Difference $H_{62/8} - H_{62/R}$ vs altitude.

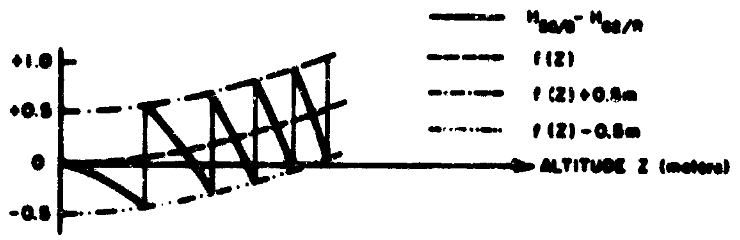


**Figure A-3** Difference $H_{36/8} - H_{62/R}$, as well as f(Z), f(Z) + 0.5 m, and f(Z) - 0.5 m all as a function of altitude.

Since $Z - H_{56/8}$ is always less than $Z - H_{62/8}$, it is apparent that $H_{56/8}$ departs from Z less rapidly than does $H_{62/8}$, and the difference $(H_{56/8} - H_{62/8}) = f(Z)$ increases from zero as Z increases from zero.

If the graphs of Figure A-1 were extended to include the region from 0 to 90 kilometers altitude, the graph of $(Z_i - H_{62/R})$ would show 1257 vertical line segments representing the same number of abrupt discontinuities while, if extended to include the region from 90 to 700 kilometers altitude, the graph would include an additional 68, 216 such discontinuities. In these same two altitude intervals a graph (not presented in Figure A-1) showing the difference between Z and rounded values of $H_{56}$, i.e., $(Z_i - H_{56/R})$ would show one less and 33 less discontinuities, respectively, than would be seen in the extended graph of $Z_i - H_{62/R}$.

If the hypothetical graph $(Z - H_{62/8})$ and the realistic graph $(Z_i - H_{62/R})$ are compared, we find that at the particular points for which Z is an integral multiple of one meter, $Z - Z_i = 0$, and the difference $H_{62/8} - H_{62/R})$ varies between +0.5 and about -0.5 meter in accordance with the graph of Figure A-2. A careful examination shows that the limiting differences at the points of discontinuity are separated by values of 0.999 m', 0.999 m', or 0.99 m', consistent with the fixed eight significant figures in $H_{62/8}$ and the varying number in $H_{62/8}$ depending upon the altitude region. It is also apparent that these discontinuities are symmetrical about the horizontal axis.

A comparison of the discontinuous graph $(Z_i - H_{62/R})$ and the continuous graph $(Z - H_{56/8})$ at those points for which Z has an integral value yields a difference $(H_{56/8} - H_{62/R})$ which follows the pattern of Figure A-3. In this figure, the discontinuities occur at the same altitudes as for Figure A-2. The limiting differences at the points of discontinuity are the same as for the corresponding discontinuity of Figure A-2. Contrary to the situation in Figure A-2, these limiting points are not symmetrical about the horizontal axis but are symmetrical about the curved-line function f(Z) which is the function being sought for use in Equation (A-2).

Unfortunately, the number of values of $H_{62/R}$ in the U.S. Standard Atmosphere is not sufficient so that a graph of these points would show the detailed type of pattern of Figure A-3: there are only 420 values of $H_{62/R}$ for altitudes between 90 to 700 km compared with 68216 regions of discontinuity for that same range of altitudes. Consequently, any single value $(H_{56/8} - H_{62/R})$ from available data represents but a single point on a section of graph which, on the average, might have 170 regions of discontinuity. Obviously the detailed step-function graph cannot be produced from the available data. On a graph in which the altitude scale has been sufficiently compressed, however, a plot of the available 420 data points (above 90 km) appears as a band of randomly scattered points in which the extreme ordinate values for any single value of abscissa may never exceed a difference corresponding to one geopotential meter, as in Figure A-3. Hopefully, a sufficient number of data points lie near enough to the two extremes to permit the establishment of a good approximation to the locus of the upper and lower boundary of the 1-m' band.

The smoothed lower bound of the envelope of the set of scattered data $(H_{56/8} - H_{62/R})$ represents a subset of values which are never more than 0.4999 m′ below the desired function $(H_{56/8} - H_{62/8}) = f(Z)$. Similarly, the smoothed upper bound of the envelope of the scattered data represents a set of values which are never more than 0.5 m′ greater than the desired f (Z). Thus, if 0.4999 m′, 0.499 m′, or 0.49 m′ is added to the individual values of the difference set $(H_{56/8} - H_{62/R})$, each in the appropriate altitude region, a set of data called Source Set 1 is formed. The smoothed lower bound of the values of Source Set 1 consist of a smaller set of data called Subset 1. The points of Subset 1 will have values equal to or very slightly greater than the values of the desired function f(Z) at the corresponding altitudes.

Similarly if 0.5000 m′ is subtracted from the individual values of the difference set $(H_{56/8} - H_{62/R})$, to form Source Set 2, the smoothed upper bound of these downwardly adjusted data form a set of points called Subset 2. These data points of Subset 2 will have values equal to or slightly less than the values of the desired function f(Z).

The points of Subset 1 and Subset 2 are then subjected to a further graphical selection to form Smoothed Subsets 1 and 2. The points of these Smoothed Subsets 1 and 2 are then combined, and when processed in a curve-fitting program determine a close approximation to the desired function f(Z). The numerical and graphical processes employed depend upon the assumption that the function f(Z) as well as its first derivative are monotonically increasing with Z, a situation which is apparently true for the upper and lower bounds of the band of data points.

The detailed steps of the process are as follows:

(1) Prepare Source Set 1 by performing the following operations as appropriate:

add 0.4999 to $(H_{56/8} - H_{62/R})$ for $1.0000 \text{ km} \leq Z \leq 9.9999$ km,

add 0.499 to $(H_{56/8} - H_{62/R})$ for $10.000 \text{ km} \leq Z \leq 99.999$ km,

add 0.49 to $(H_{56/8} - H_{62/R})$ for $100.00 \text{ km} \leq Z \leq 999.99$ km,

(2) Prepare Subset 1 as follows:

(a) Scan the entire Source Set 1 for the smallest positive number (all of the numbers of this set will be positive), and store this value with its corresponding value of Z as the first entry of Subset 1.

(b) Remove this value and those for lower altitudes from Source Set 1 and discard.

(c) Scan the remaining members of Source Set 1 for the lowest value and store this value with its corresponding value of Z as the 2nd entry of Subset 1.

(d) Remove this member along with those associated with lower altitudes from Source Set 1. Repeat steps (c) and (d) until all the values of Source Set 1 have been removed, and Subset 1 has been developed. If there are two or more lowest values at any of the above steps store only that one corresponding to the greatest altitude, and reject the others.

(3) Punch and print the stored values of Subset 1. This is presented as column 5 DIF+INC, and column 1 in Table A-2 which consists of an ordered listing of the appropriate cards extracted from Source Set 1.

(4) Prepare Source Set 2 by performing the folloing operation for all altitudes of interest:

$$\text{subtract } 0.5000 \text{ from } (H_{56/8} - H_{62/R})$$

(5) Prepare Subset 2 as follows:

(a) Scan the entire Source Set 2 for negative values, which will be found at the low-altitude end of the set, removing and discarding these members of the set.

(b) Scan the remainder of Source Set 2 for the largest positive value and store this value along with its associated altitude value as the first member of Subset 2.

(c) Remove this member from Source Set 2 along with all members associated with greater altitudes.

(d) Repeat steps (b) and (c) until all members of Source Set 2 have been removed and Subset 2 has been developed. If there are two or more greatest values at any point in the scanning operation, store only that one associated with the lowest altitude and discard the others.

(6) Punch and print the stored values of Subset 2. This is presented as column 6 DIF-0.5, and column 1 in Table A-3 which consists of an ordered listing of the appropriate cards extracted from Source Set 2.

(7) Plot the data points of Subset 1 on large-scale graphs (not shown in this report) for further data selection. The suggested scales for these graphs are indicated in Table A-4.

(8) Select certain points of Subset 1 which appear to form a smooth monotonically increasing lower bound to the total of Subset 1 data. These selected points comprise Smooth Subset 1 and are those points which may be connected sequentially with straight-line segments meeting the following two conditions:

(a) Each of these line segments lies below all those points in Subset 1 having altitude values within the altitude interval encompassed by the particular line segment.

(b) The successive line segments have slopes which are monotonically increasing for increasing altitudes.

## TABLE A-2

VALUES OF SUBSET 1 AS A FUNCTION OF GEOMETRIC ALTITUDE
WITH THE RELATED VALUES OF H62/R, H56/8;
AND THE DIFFERENCES H56/8 - H62/R

| GEOMETRIC ALTITUDE METERS | H62/R GEOPOTENTIAL METERS | H56/8 GEOPOTENTIAL METERS | DIF H56-H62 GEOPT. METERS | DIF+INC GEOPT. METERS | DIF-0.5 GEOPT. METERS |
|---|---|---|---|---|---|
| 31000. | .30850000E 05 | .30849557E 05 | -.4430 | .0560 | -.9430 |
| 58000. | .57476000E 05 | .57475585E 05 | -.4150 | .0840 | -.9150 |
| 82000. | .80956000E 05 | .80955730E 05 | -.3000 | .1990 | -.8000 |
| 96000. | .94572000E 05 | .94571776E 05 | -.2240 | .2750 | -.7240 |
| 105000. | .10329400E 06 | .10329381E 06 | -.1900 | .3000 | -.6900 |
| 111000. | .10909500E 06 | .10909501E 06 | .0100 | .5000 | -.4900 |
| 140000. | .13698300E 06 | .13698311E 06 | .1100 | .6000 | -.3900 |
| 144000. | .14081000E 06 | .14081022E 06 | .2200 | .7100 | -.2800 |
| 145000. | .14176600E 06 | .14176626E 06 | .2600 | .7500 | -.2400 |
| 152000. | .14845000E 06 | .14845032E 06 | .3200 | .8100 | -.1800 |
| 156000. | .15226300E 06 | .15226332E 06 | .3300 | .8200 | -.1700 |
| 166000. | .16177500E 06 | .16177542E 06 | .4200 | .9100 | -.0800 |
| 173000. | .16841600E 06 | .16841652E 06 | .5200 | 1.0100 | .0200 |
| 179000. | .17409700E 06 | .17409759E 06 | .5900 | 1.0800 | .0900 |
| 190000. | .18448500E 06 | .18448582E 06 | .8200 | 1.3100 | .3200 |
| 192000. | .18637000E 06 | .18637085E 06 | .8500 | 1.3400 | .3500 |
| 193000. | .18731200E 06 | .18731292E 06 | .9200 | 1.4100 | .4200 |
| 209000. | .20234600E 06 | .20234716E 06 | 1.1600 | 1.6500 | .6600 |
| 213000. | .20609300E 06 | .20609428E 06 | 1.2800 | 1.7700 | .7800 |
| 224000. | .21637400E 06 | .21637536E 06 | 1.3600 | 1.8500 | .8600 |
| 228000. | .22010400E 06 | .22010541E 06 | 1.4100 | 1.9000 | .9100 |
| 233000. | .22476000E 06 | .22476162E 06 | 1.6200 | 2.1100 | 1.1200 |
| 239000. | .23033800E 06 | .23033975E 06 | 1.7500 | 2.2400 | 1.2500 |
| 243000. | .23405100E 06 | .23405286E 06 | 1.8600 | 2.3500 | 1.3600 |
| 252000. | .24238900E 06 | .24239094E 06 | 1.9400 | 2.4300 | 1.4400 |
| 253000. | .24331400E 06 | .24331600E 06 | 2.0000 | 2.4900 | 1.5000 |
| 255000. | .24516300E 06 | .24516525E 06 | 2.2500 | 2.7400 | 1.7500 |
| 261000. | .25070400E 06 | .25070634E 06 | 2.3400 | 2.8300 | 1.8400 |
| 267000. | .25623500E 06 | .25623739E 06 | 2.3900 | 2.8800 | 1.8900 |
| 275000. | .26359400E 06 | .26359656E 06 | 2.5600 | 3.0500 | 2.0600 |
| 278000. | .26634900E 06 | .26635165E 06 | 2.6500 | 3.1400 | 2.1500 |
| 283000. | .27093500E 06 | .27093798E 06 | 2.9800 | 3.4700 | 2.4800 |
| 287000. | .27459900E 06 | .27460206E 06 | 3.0600 | 3.5500 | 2.5600 |
| 291000. | .27734400E 06 | .27734722E 06 | 3.2200 | 3.7100 | 2.7200 |
| 298000. | .28465200E 06 | .28465558E 06 | 3.5800 | 4.0700 | 3.0800 |
| 299000. | .28556400E 06 | .28556788E 06 | 3.8800 | 4.3700 | 3.3800 |
| 300000. | .28647600E 06 | .28647992E 06 | 3.9200 | 4.4100 | 3.4200 |
| 312000. | .29739900E 06 | .29740300E 06 | 4.0000 | 4.4900 | 3.5000 |
| 320000. | .30465900E 06 | .30466323E 06 | 4.2300 | 4.7200 | 3.7300 |
| 322000. | .30647100E 06 | .30647558E 06 | 4.5800 | 5.0700 | 4.0800 |
| 336000. | .31912700E 06 | .31913164E 06 | 4.6400 | 5.1300 | 4.1400 |
| 344000. | .32633500E 06 | .32633993E 06 | 4.9300 | 5.4200 | 4.4300 |

## TABLE A-2 (Continued)

| GEOMETRIC ALTITUDE METERS | H62/R GEOPOTENTIAL METERS | H56/S GEOPOTENTIAL METERS | DIF H56-H62 GEOPT. METERS | DIF+INC GEOPT. METERS | DIF-0.5 GEOPT. METERS |
|---|---|---|---|---|---|
| 346000. | .32813400E 06 | .32813931E 06 | 5.3100 | 5.8000 | 4.8100 |
| 348000. | .32993200E 06 | .32993763E 06 | 5.6300 | 6.1200 | 5.1300 |
| 362000. | .34249000E 06 | .34249582E 06 | 5.8200 | 6.3100 | 5.3200 |
| 368000. | .34785600E 06 | .34786190E 06 | 5.9000 | 6.3900 | 5.4000 |
| 380000. | .35855900E 06 | .35856539E 06 | 6.3900 | 6.8800 | 5.8900 |
| 382000. | .36033900E 06 | .36034559E 06 | 6.5900 | 7.0800 | 6.0900 |
| 384000. | .36211800E 06 | .36212473E 06 | 6.7300 | 7.2200 | 6.2300 |
| 386000. | .36389600E 06 | .36390284E 06 | 6.8400 | 7.3300 | 6.3400 |
| 390000. | .36744900E 06 | .36745586E 06 | 6.8600 | 7.3500 | 6.3600 |
| 398000. | .37454200E 06 | .37454930E 06 | 7.3000 | 7.7900 | 6.8000 |
| 402000. | .37808200E 06 | .37808971E 06 | 7.7100 | 8.2000 | 7.2100 |
| 406000. | .38161800E 06 | .38162595E 06 | 7.9500 | 8.4400 | 7.4500 |
| 410000. | .38515000E 06 | .38515801E 06 | 8.0100 | 8.5000 | 7.5100 |
| 416000. | .39044000E 06 | .39044826E 06 | 8.2600 | 8.7500 | 7.7600 |
| 418000. | .39220100E 06 | .39220959E 06 | 8.5900 | 9.0800 | 8.0900 |
| 420000. | .39396100E 06 | .39396988E 06 | 8.8800 | 9.3700 | 8.3800 |
| 422000. | .39572000E 06 | .39572915E 06 | 9.1500 | 9.6400 | 8.6500 |
| 424000. | .39747800E 06 | .39748736E 06 | 9.3600 | 9.8500 | 8.8600 |
| 426000. | .39923500E 06 | .39924454E 06 | 9.5400 | 10.0300 | 9.0400 |
| 428000. | .40099100E 06 | .40100068E 06 | 9.6800 | 10.1700 | 9.1800 |
| 442000. | .41325500E 06 | .41326479E 06 | 9.7900 | 10.2800 | 9.2900 |
| 450000. | .42024300E 06 | .42025319E 06 | 10.1900 | 10.6800 | 9.6900 |
| 454000. | .42372600E 06 | .42373674E 06 | 10.7400 | 11.2300 | 10.2400 |
| 460000. | .42894800E 06 | .42895889E 06 | 10.8900 | 11.3800 | 10.3900 |
| 464000. | .43242400E 06 | .43243521E 06 | 11.2100 | 11.7000 | 10.7100 |
| 468000. | .43589600E 06 | .43590747E 06 | 11.4700 | 11.9600 | 10.9700 |
| 472000. | .43936400E 06 | .43937566E 06 | 11.6600 | 12.1500 | 11.1600 |
| 476000. | .44282800E 06 | .44283978E 06 | 11.7800 | 12.2700 | 11.2800 |
| 478000. | .44455800E 06 | .44457032E 06 | 12.3200 | 12.8100 | 11.8200 |
| 482000. | .44801600E 06 | .44802837E 06 | 12.3700 | 12.8600 | 11.8700 |
| 488000. | .45319500E 06 | .45320787E 06 | 12.8700 | 13.3600 | 12.3700 |
| 494000. | .45836500E 06 | .45837829E 06 | 13.2900 | 13.7800 | 12.7900 |
| 500000. | .46352600E 06 | .46353966E 06 | 13.6600 | 14.1500 | 13.1600 |
| 506000. | .46867800E 06 | .46869201E 06 | 14.0100 | 14.5000 | 13.5100 |
| 512000. | .47382100E 06 | .47383536E 06 | 14.3600 | 14.8500 | 13.8600 |
| 514000. | .47553300E 06 | .47554781E 06 | 14.8100 | 15.3000 | 14.3100 |
| 520000. | .48066400E 06 | .48067919E 06 | 15.1900 | 15.6800 | 14.6900 |
| 526000. | .48578600E 06 | .48580162E 06 | 15.6200 | 16.1100 | 15.1200 |
| 528000. | .48749100E 06 | .48750711E 06 | 16.1100 | 16.6000 | 15.6100 |
| 532000. | .49089900E 06 | .49091514E 06 | 16.1400 | 16.6300 | 15.6400 |
| 536000. | .49430300E 06 | .49431920E 06 | 16.2000 | 16.6900 | 15.7000 |
| 538000. | .49600300E 06 | .49601975E 06 | 16.7500 | 17.2400 | 16.2500 |
| 542000. | .49940100E 06 | .49941790E 06 | 16.9000 | 17.3900 | 16.4000 |
| 546000. | .50279500E 06 | .50281209E 06 | 17.0900 | 17.5800 | 16.5900 |
| 550000. | .50618500E 06 | .50620237E 06 | 17.3700 | 17.8600 | 16.8700 |
| 554000. | .50957100E 06 | .50958872E 06 | 17.7200 | 18.2100 | 17.2200 |

**TABLE A-2 (Continued)**

| GEOMETRIC ALTITUDE | H62/R | H56/8 | DIF H56-H62 | DIF+INC | DIF-0.5 |
|---|---|---|---|---|---|
| METERS | GEOPOTENTIAL METERS | GEOPOTENTIAL METERS | GEOPT. METERS | GEOPT. METERS | GEOPT. METERS |
| 558000. | .51295300E 06 | .51297114E 06 | 18.1400 | 18.6300 | 17.6400 |
| 562000. | .51633100E 06 | .51634966E 06 | 18.6600 | 19.1500 | 18.1600 |
| 568000. | .52139100E 06 | .52141012E 06 | 19.1200 | 19.6100 | 18.6200 |
| 578000. | .52980500E 06 | .52982475E 06 | 19.7500 | 20.2400 | 19.2500 |
| 580000. | .53148400E 06 | .53150478E 06 | 20.7800 | 21.2700 | 20.2800 |
| 582000. | .53316300E 06 | .53318382E 06 | 20.8200 | 21.3100 | 20.3200 |
| 584000. | .53484100E 06 | .53486190E 06 | 20.9000 | 21.3900 | 20.4000 |
| 586000. | .53651800E 06 | .53653902E 06 | 21.0200 | 21.5100 | 20.5200 |
| 588000. | .53819400E 06 | .53821517E 06 | 21.1700 | 21.6600 | 20.6700 |
| 590000. | .53986900E 06 | .53989035E 06 | 21.3500 | 21.8400 | 20.8500 |
| 592000. | .54154300E 06 | .54156457E 06 | 21.5700 | 22.0600 | 21.0700 |
| 600000. | .54823000E 06 | .54825182E 06 | 21.8200 | 22.3100 | 21.3200 |
| 602000. | .54989900E 06 | .54992122E 06 | 22.2200 | 22.7100 | 21.7200 |
| 604000. | .55156700E 06 | .55158968E 06 | 22.6800 | 23.1700 | 22.1800 |
| 608000. | .55490100E 06 | .55492370E 06 | 22.7000 | 23.1900 | 22.2000 |
| 610000. | .55656600E 06 | .55658928E 06 | 23.2800 | 23.7700 | 22.7800 |
| 614000. | .55989400E 06 | .55991756E 06 | 23.5600 | 24.0500 | 23.0600 |
| 620000. | .56487900E 06 | .56490284E 06 | 23.8400 | 24.3300 | 23.3400 |
| 626000. | .56985500E 06 | .56987954E 06 | 24.5400 | 25.0300 | 24.0400 |
| 628000. | .57151100E 06 | .57153654E 06 | 25.5400 | 26.0300 | 25.0400 |
| 630000. | .57316700E 06 | .57319260E 06 | 25.6000 | 26.0900 | 25.1000 |
| 632000. | .57482200E 06 | .57484770E 06 | 25.7000 | 26.1900 | 25.2000 |
| 634000. | .57647600E 06 | .57650186E 06 | 25.8600 | 26.3500 | 25.3600 |
| 636000. | .57812900E 06 | .57815508E 06 | 26.0800 | 26.5700 | 25.5800 |
| 638000. | .57978100E 06 | .57980734E 06 | 26.3400 | 26.8300 | 25.8400 |
| 644000. | .58473200E 06 | .58475848E 06 | 26.4800 | 26.9700 | 25.9800 |
| 646000. | .58638000E 06 | .58640697E 06 | 26.9700 | 27.4600 | 26.4700 |
| 650000. | .58967400E 06 | .58970114E 06 | 27.1400 | 27.6300 | 26.6400 |
| 654000. | .59296400E 06 | .59299155E 06 | 27.5500 | 28.0400 | 27.0500 |
| 660000. | .59789200E 06 | .59792012E 06 | 28.1200 | 28.6100 | 27.6200 |
| 662000. | .59953200E 06 | .59956110E 06 | 29.1000 | 29.5900 | 28.6000 |
| 664000. | .60117200E 06 | .60120115E 06 | 29.1500 | 29.6400 | 28.6500 |
| 666000. | .60281100E 06 | .60284028E 06 | 29.2800 | 29.7700 | 28.7800 |
| 668000. | .60444900E 06 | .60447845E 06 | 29.4500 | 29.9400 | 28.9500 |
| 670000. | .60608600E 06 | .60611570E 06 | 29.7000 | 30.1900 | 29.2000 |
| 672000. | .60772200E 06 | .60775203E 06 | 30.0300 | 30.5200 | 29.5300 |
| 678000. | .61262500E 06 | .61265539E 06 | 30.3900 | 30.8800 | 29.8900 |
| 682000. | .61588900E 06 | .61591966E 06 | 30.6600 | 31.1500 | 30.1600 |
| 686000. | .61914900E 06 | .61918023E 06 | 31.2300 | 31.7200 | 30.7300 |
| 690000. | .62240500E 06 | .62243709E 06 | 32.0900 | 32.5800 | 31.5900 |
| 692000. | .62403200E 06 | .62406414E 06 | 32.1400 | 32.6300 | 31.6400 |
| 694000. | .62565800E 06 | .62569026E 06 | 32.2600 | 32.7500 | 31.7600 |
| 696000. | .62728300E 06 | .62731545E 06 | 32.4500 | 32.9400 | 31.9500 |
| 698000. | .62890700E 06 | .62893974E 06 | 32.7400 | 33.2300 | 32.2400 |
| 700000. | .63053000E 06 | .63056309E 06 | 33.0900 | 33.5800 | 32.5900 |

**TABLE A-3**

VALUES OF SUBSET 2 AS A FUNCTION OF GEOMETRIC ALTITUDE
WITH THE RELATED VALUES OF H62/R, H56/8,
AND THE DIFFERENCES H56/8 - H62/R

| GEOMETRIC ALTITUDE METERS | H62/R GEOPOTENTIAL METERS | H56/8 GEOPOTENTIAL METERS | DIF H56-H62 GEOPT. METERS | DIF+INC GEOPT. METERS | DIF-0.5 GEOPT. METERS |
|---|---|---|---|---|---|
| 700000. | .63053000E 06 | .63056309E 06 | 33.0900 | 33.5800 | 32.5900 |
| 698000. | .62890700E 06 | .62893974E 06 | 32.7400 | 33.2300 | 32.2400 |
| 696000. | .62728300E 06 | .62731545E 06 | 32.4500 | 32.9400 | 31.9500 |
| 694000. | .62565800E 06 | .62569026E 06 | 32.2600 | 32.7500 | 31.7600 |
| 692000. | .62403200E 06 | .62406414E 06 | 32.1400 | 32.6300 | 31.6400 |
| 688000. | .62077700E 06 | .62080912E 06 | 32.1200 | 32.6100 | 31.6200 |
| 684000. | .61751900E 06 | .61755040E 06 | 31.4000 | 31.8900 | 30.9000 |
| 680000. | .61425700E 06 | .61428800E 06 | 31.0000 | 31.4900 | 30.5000 |
| 676000. | .61099100E 06 | .61102186E 06 | 30.8600 | 31.3500 | 30.3600 |
| 674000. | .60935700E 06 | .60938741E 06 | 30.4100 | 30.9000 | 29.9100 |
| 672000. | .60772200E 06 | .60775203E 06 | 30.0300 | 30.5200 | 29.5300 |
| 670000. | .60608600E 06 | .60611570E 06 | 29.7000 | 30.1900 | 29.2000 |
| 668000. | .60444900E 06 | .60447845E 06 | 29.4500 | 29.9400 | 28.9500 |
| 666000. | .60281100E 06 | .60284028E 06 | 29.2800 | 29.7700 | 28.7800 |
| 664000. | .60117200E 06 | .60120115E 06 | 29.1500 | 29.6400 | 28.6500 |
| 662000. | .59953200E 06 | .59956110E 06 | 29.1000 | 29.5900 | 28.6000 |
| 656000. | .59460700E 06 | .59463534E 06 | 28.3400 | 28.8300 | 27.8400 |
| 652000. | .59131900E 06 | .59134681E 06 | 27.8100 | 28.3000 | 27.3100 |
| 648000. | .58802700E 06 | .58805453E 06 | 27.5300 | 28.0200 | 27.0300 |
| 642000. | .58308200E 06 | .58310905E 06 | 27.0500 | 27.5400 | 26.5500 |
| 640000. | .58143200E 06 | .58145866E 06 | 26.6600 | 27.1500 | 26.1600 |
| 638000. | .57978100E 06 | .57980734E 06 | 26.3400 | 26.8300 | 25.8400 |
| 636000. | .57812900E 06 | .57815508E 06 | 26.0800 | 26.5700 | 25.5800 |
| 634000. | .57647600E 06 | .57650186E 06 | 25.8600 | 26.3500 | 25.3600 |
| 632000. | .57482200E 06 | .57484770E 06 | 25.7000 | 26.1900 | 25.2000 |
| 630000. | .57316700E 06 | .57319260E 06 | 25.6000 | 26.0900 | 25.1000 |
| 628000. | .57151100E 06 | .57153654E 06 | 25.5400 | 26.0300 | 25.0400 |
| 622000. | .56653800E 06 | .56656270E 06 | 24.7000 | 25.1900 | 24.2000 |
| 616000. | .56155600E 06 | .56158028E 06 | 24.2800 | 24.7700 | 23.7800 |
| 612000. | .55823000E 06 | .55825390E 06 | 23.9000 | 24.3900 | 23.4000 |
| 610000. | .55656600E 06 | .55658928E 06 | 23.2800 | 23.7700 | 22.7800 |
| 606000. | .55323400E 06 | .55325717E 06 | 23.1700 | 23.6600 | 22.6700 |
| 604000. | .55156700E 06 | .55158968E 06 | 22.6800 | 23.1700 | 22.1800 |
| 598000. | .54655900E 06 | .54658145E 06 | 22.4500 | 22.9400 | 21.9500 |
| 596000. | .54488800E 06 | .54491011E 06 | 22.1100 | 22.6000 | 21.6100 |
| 594000. | .54321600E 06 | .54323782E 06 | 21.8200 | 22.3100 | 21.3200 |
| 592000. | .54154300E 06 | .54156457E 06 | 21.5700 | 22.0600 | 21.0700 |
| 590000. | .53986900E 06 | .53989035E 06 | 21.3500 | 21.8400 | 20.8500 |
| 588000. | .53819400E 06 | .53821517E 06 | 21.1700 | 21.6600 | 20.6700 |
| 586000. | .53651800E 06 | .53653962E 06 | 21.0200 | 21.5100 | 20.5200 |
| 584000. | .53484100E 06 | .53486190E 06 | 20.9000 | 21.3900 | 20.4000 |
| 582000. | .53316300E 06 | .53318382E 06 | 20.8200 | 21.3100 | 20.3200 |

**TABLE A-3 (Continued)**

| GEOMETRIC ALTITUDE METERS | H62/R GEOPOTENTIAL METERS | H56/8 GEOPOTENTIAL METERS | DIF H56-H62 GEOPT. METERS | DIF+INC GEOPT. METERS | DIF-0.5 GEOPT. METERS |
|---|---|---|---|---|---|
| 580000. | .53148400E 06 | .53150478E 06 | 20.7800 | 21.2700 | 20.2800 |
| 570000. | .52307500E 06 | .52309499E 06 | 19.9900 | 20.4800 | 19.4900 |
| 564000. | .51801800E 06 | .51803745E 06 | 19.4500 | 19.9400 | 18.9500 |
| 560000. | .51464200E 06 | .51466090E 06 | 18.9000 | 19.3900 | 18.4000 |
| 556000. | .51126200E 06 | .51128042E 06 | 18.4200 | 18.9100 | 17.9200 |
| 552000. | .50787800E 06 | .50789603E 06 | 18.0300 | 18.5200 | 17.5300 |
| 548000. | .50449000E 06 | .50450773E 06 | 17.7300 | 18.2200 | 17.2300 |
| 544000. | .50109800E 06 | .50111548E 06 | 17.4800 | 17.9700 | 16.9800 |
| 540000. | .49770200E 06 | .49771931E 06 | 17.3100 | 17.8000 | 16.8100 |
| 538000. | .49600300E 06 | .49601975E 06 | 16.7500 | 17.2400 | 16.2500 |
| 534000. | .49260100E 06 | .49261766E 06 | 16.6600 | 17.1500 | 16.1600 |
| 530000. | .48919500E 06 | .48921163E 06 | 16.6300 | 17.1200 | 16.1300 |
| 524000. | .48407900E 06 | .48409514E 06 | 16.1400 | 16.6300 | 15.6400 |
| 518000. | .47895400E 06 | .47896973E 06 | 15.7300 | 16.2200 | 15.2300 |
| 516000. | .47724400E 06 | .47725927E 06 | 15.2700 | 15.7600 | 14.7700 |
| 510000. | .47210700E 06 | .47212191E 06 | 14.9100 | 15.4000 | 14.4100 |
| 504000. | .46696100E 06 | .46697557E 06 | 14.5700 | 15.0600 | 14.0700 |
| 498000. | .46180600E 06 | .46182021E 06 | 14.2100 | 14.7000 | 13.7100 |
| 492000. | .45664200E 06 | .45665583E 06 | 13.8300 | 14.3200 | 13.3300 |
| 486000. | .45146900E 06 | .45148238E 06 | 13.3800 | 13.8700 | 12.8800 |
| 484000. | .44974300E 06 | .44975587E 06 | 12.8700 | 13.3600 | 12.3700 |
| 480000. | .44628700E 06 | .44629986E 06 | 12.8600 | 13.3500 | 12.3600 |
| 478000. | .44455800E 06 | .44457032E 06 | 12.3200 | 12.8100 | 11.8200 |
| 474000. | .44109600E 06 | .44110823E 06 | 12.2300 | 12.7200 | 11.7300 |
| 470000. | .43763000E 06 | .43764207E 06 | 12.0700 | 12.5600 | 11.5700 |
| 466000. | .43416000E 06 | .43417186E 06 | 11.8600 | 12.3500 | 11.3600 |
| 462000. | .43068600E 06 | .43069756E 06 | 11.5600 | 12.0500 | 11.0600 |
| 456000. | .42546700E 06 | .42547848E 06 | 11.4800 | 11.9700 | 10.9800 |
| 452000. | .42198300E 06 | .42199397E 06 | 10.9700 | 11.4600 | 10.4700 |
| 444000. | .41500200E 06 | .41501268E 06 | 10.6800 | 11.1700 | 10.1800 |
| 436000. | .40800500E 06 | .40801494E 06 | 9.9400 | 10.4300 | 9.4400 |
| 434000. | .40625300E 06 | .40626292E 06 | 9.9200 | 10.4100 | 9.4200 |
| 432000. | .40450000E 06 | .40450988E 06 | 9.8800 | 10.3700 | 9.3800 |
| 430000. | .40274600E 06 | .40275580E 06 | 9.8000 | 10.2900 | 9.3000 |
| 428000. | .40099100E 06 | .40100068E 06 | 9.6800 | 10.1700 | 9.1800 |
| 426000. | .39923500E 06 | .39924454E 06 | 9.5400 | 10.0300 | 9.0400 |
| 424000. | .39747800E 06 | .39748736E 06 | 9.3600 | 9.8500 | 8.8600 |
| 422000. | .39572000E 06 | .39572915E 06 | 9.1500 | 9.6400 | 8.6500 |
| 420000. | .39396100E 06 | .39396988E 06 | 8.8800 | 9.3700 | 8.3800 |
| 414000. | .38867700E 06 | .38868587E 06 | 8.8700 | 9.3600 | 8.3700 |
| 408000. | .38338400E 06 | .38339249E 06 | 8.4900 | 8.9800 | 7.9900 |
| 404000. | .37985000E 06 | .37985836E 06 | 8.3600 | 8.8500 | 7.8600 |
| 400000. | .37631200E 06 | .37632003E 06 | 8.0300 | 8.5200 | 7.5300 |
| 392000. | .36922200E 06 | .36923080E 06 | 7.8000 | 8.2900 | 7.3000 |
| 379000. | .35677700E 06 | .35678411E 06 | 7.1100 | 7.6000 | 6.6100 |
| 372000. | .35142700E 06 | .35143398E 06 | 6.9800 | 7.4700 | 6.4800 |

## TABLE A-3 (Continued)

| GEOMETRIC ALTITUDE METERS | H62/R GEOPOTENTIAL METERS | H56/8 GEOPOTENTIAL METERS | DIF H56-H62 GEOPT. METERS | DIF+INC GEOPT. METERS | DIF-0.5 GEOPT. METERS |
|---|---|---|---|---|---|
| 364000. | .34427900E 06 | .34428558E 06 | 6.5800 | 7.0700 | 6.0800 |
| 356000. | .33711400E 06 | .33712015E 06 | 6.1500 | 6.6400 | 5.6500 |
| 354000. | .33532000E 06 | .33532613E 06 | 6.1300 | 6.6200 | 5.6300 |
| 352000. | .33352500E 06 | .33353102E 06 | 6.0200 | 6.5100 | 5.5200 |
| 350000. | .33172900E 06 | .33173486E 06 | 5.8600 | 6.3500 | 5.3600 |
| 348000. | .32993200E 06 | .32993763E 06 | 5.6300 | 6.1200 | 5.1300 |
| 342000. | .32453400E 06 | .32453947E 06 | 5.4700 | 5.9600 | 4.9700 |
| 338000. | .32093000E 06 | .32093532E 06 | 5.3200 | 5.8100 | 4.8200 |
| 33[illegible]. | .31370900E 06 | .31371410E 06 | 5.1000 | 5.5900 | 4.6000 |
| 328000. | .31190100E 06 | .31190609E 06 | 5.0900 | 5.5800 | 4.5900 |
| 326[illegible]. | .31009200E 06 | .31009700E 06 | 5.0000 | 5.4900 | 4.5000 |
| 324000. | .30828200E 06 | .30828683E 06 | 4.8300 | 5.3200 | 4.3300 |
| 318000. | .30284500E 06 | .30284981E 06 | 4.8100 | 5.3000 | 4.3100 |
| 314000. | .29921500E 06 | .29921968E 06 | 4.6800 | 5.1700 | 4.1800 |
| 3[illegible]6[illegible]. | .29194200E 06 | .29194638E 06 | 4.3800 | 4.8700 | 3.8800 |
| 304000. | .29012100E 06 | .29012532E 06 | 4.3200 | 4.8100 | 3.8200 |
| 302[illegible]. | .28829900E 06 | .28830316E 06 | 4.1600 | 4.6500 | 3.6600 |
| 296000. | .28282600E 06 | .28283013E 06 | 4.1300 | 4.6200 | 3.6300 |
| 292000. | .27917200E 06 | .27917597E 06 | 3.9700 | 4.4600 | 3.4700 |
| 291000. | .27825800E 06 | .27826173E 06 | 3.7300 | 4.2200 | 3.2300 |
| 277000. | .26543000E 06 | .26543357E 06 | 3.5700 | 4.0600 | 3.0700 |
| 273000. | .26175500E 06 | .26175842E 06 | 3.4200 | 3.9100 | 2.9200 |
| 268000. | .25715500E 06 | .25715826E 06 | 3.2600 | 3.7500 | 2.7600 |
| 266000. | .25531300E 06 | .25531625E 06 | 3.2500 | 3.7400 | 2.7500 |
| 263000. | .25254800E 06 | .25255115E 06 | 3.1500 | 3.6400 | 2.6500 |
| 258000. | .24793400E 06 | .24793705E 06 | 3.0500 | 3.5400 | 2.5500 |
| 254000. | .24423800E 06 | .24424077E 06 | 2.7700 | 3.2600 | 2.2700 |
| 245000. | .23590500E 06 | .23590774E 06 | 2.7400 | 3.2300 | 2.2400 |
| 237000. | .22847900E 06 | .22848149E 06 | 2.4900 | 2.9800 | 1.9900 |
| 227000. | .21917100E 06 | .21917333E 06 | 2.3300 | 2.8200 | 1.8300 |
| 225000. | .21730600E 06 | .21730830E 06 | 2.3000 | 2.7900 | 1.8000 |
| 223000. | .21544000E 06 | .21544213E 06 | 2.1300 | 2.6200 | 1.6300 |
| 215000. | .20796400E 06 | .20796612E 06 | 2.1200 | 2.6100 | 1.6200 |
| 2[illegible]3000. | .19671600E 06 | .19671792E 06 | 1.9200 | 2.4100 | 1.4200 |
| 194000. | .18825300E 06 | .18825472E 06 | 1.7200 | 2.2100 | 1.2200 |
| 188000. | .18259800E 06 | .18259965E 06 | 1.6500 | 2.1400 | 1.1500 |
| 179000. | .17315000E 06 | .17315146E 06 | 1.4600 | 1.9500 | .9600 |
| 175000. | .17031000E 06 | .17031138E 06 | 1.3800 | 1.8700 | .8800 |
| 16[illegible]. | .16462200E 06 | .16462336E 06 | 1.3600 | 1.8500 | .8600 |
| 164000. | .15987400E 06 | .15987533E 06 | 1.3300 | 1.8200 | .8300 |
| 162000. | .15797300E 06 | .15797409E 06 | 1.0900 | 1.5800 | .5900 |
| 158000. | .15416700E 06 | .15416808E 06 | 1.0800 | 1.5700 | .5800 |
| 150000. | .14654100E 06 | .14654206E 06 | 1.0600 | 1.5500 | .5600 |
| 146000. | .14272100E 06 | .14272201E 06 | 1.0100 | 1.5000 | .5100 |
| 143000. | .13985300E 06 | .13985388E 06 | .8800 | 1.3700 | .3800 |
| 131000. | .12835400E 06 | .12835496E 06 | .8600 | 1.3500 | .3600 |

## TABLE A-3 (Continued)

| GEOMETRIC ALTITUDE | H62/R | H56/8 | DIF H56-H62 | DIF+INC | DIF-0.5 |
|---|---|---|---|---|---|
| METERS | GEOPOTENTIAL METERS | GEOPOTENTIAL METERS | GEOPT. METERS | GEOPT. METERS | GEOPT. METERS |
| 112000. | .11006000E 06 | .11006083E 06 | .8300 | 1.3200 | .3300 |
| 108000. | .10619500E 06 | .10619575E 06 | .7500 | 1.2400 | .2500 |
| 107000. | .10522800E 06 | .10522874E 06 | .7400 | 1.2300 | .2400 |
| 91000. | .89715000E 05 | .89715680E 05 | .6800 | 1.1790 | .1800 |
| 72000. | .71193000E 05 | .71193624E 05 | .6240 | 1.1230 | .1240 |
| 70000. | .69237000E 05 | .69237564E 05 | .5640 | 1.0630 | .0640 |
| 44000. | .43697000E 05 | .43697536E 05 | .5360 | 1.0350 | .0360 |

TABLE A-4

SCALE VALUES OF $f_1(Z)$ AND Z EMPLOYED IN DIFFERENT PORTIONS
OF THE GRAPHS OF SUBJECTS 1 AND 2

| Altitude Interval km | Meters of f (Z) Per 1 cm of Graph | km of Altitude Z Per 1 cm of Graph |
|---|---|---|
| 0 - 150 | 0.02 | 4 |
| 100 - 250 | 0.04 | 4 |
| 220 - 370 | 0.10 | 4 |
| 360 - 550 | 0.10 | 4 |
| 550 - 700 | 0.10 | 1 |

The series of straight-line segments meeting these conditions is designated as f(SS-1). The ordinate values of the desired function f(Z) may be equal to or less than the ordinate values of the end points of the segments of f(SS-1), but will always be less than the ordinate values of all other parts of f(SS-1).

(9) Plot the data from Subset 2 on the same graphs with Subset 1.

(10) Select certain points of Subset 2 which appear to form a smooth monotonically increasing upper bound to the total of Subset 2 data. These selected points comprise Smooth Subset 2 and are those points which may be connected sequentially with straight-line segments meeting the following two conditions:

(a) Each of these line segments lies above all those points of Subset 2 having altitude values within the altitude interval encompassed by the particular line segment.

(b) Same as condition b under step 8.

This series of straight-line segments associated with Smooth Subset 2 data is designated as f(SS-2) and will be seen to be close to but below the segments f(SS-1) prepared in Step 8. The ordinate values of the desired function f(Z) may be equal to or greater than the corresponding ordinate values of f(SS-2) for all values of Z, but there is a small probability that the ordinate values of f(Z) may sometimes be slightly less than the corresponding ordinate values of f(SS-2) for some values of Z in between the end points of some of the line segments, i.e., the points of smooth Subset 2. For any particular set of abscissa values $Z_2$ corresponding to the abscissa of the data points in Smooth Subset 2, the desired smooth function f(Z) has ordinate values which are equal to or greater than the ordinate values of the related points of Subset 2, but which are less than the corresponding ordinate values of f(SS-1). Thus, the value of f(Z) corresponding to each member of the set of ordinates $Z_2$ is bounded within small limits, and it is possible to estimate the value of f(Z) for each value of $Z_2$ to be midway between the appropriate limits. It is also possible to estimate the value of $\delta f(Z)$, the range of uncertainty of f(Z), to be equal to the separation between the above specified boundaries.

Values of f(Z) and $\delta f(Z)$ may similarly be made for the set of abscissa values Z corresponding to the abscissas of the data points in Smooth Subset 1. In these instances, however, the ordinate values of f(Z) will be equal to or less than the ordinate values of the related data points of Smooth Subset 1, and may be as low as or even slightly lower than the ordinate value of the corresponding points of f(SS-2).

(11) From the graphical representations f(SS-1) and f(SS-2) connecting the data points of Smooth Subsets 1 and 2 respectively, estimate values of f(Z) and $\delta f(Z)$ for the abscissas in the set $Z_1$ and $Z_2$ in accordance with the discussion under step 10.

The set of graphically estimated values of f(Z) and uncertainty of f(Z) in the form of 100 $\delta f(Z)/f(Z)$ are presented as a function of Z in Table A-5.

## The f(Z) Polynomial

The data of Table A-5 were fed into a digital-machine curve-fitting program designed to yield the coefficients of best-fit, first-, second-, third-, and fourth-degree polynomials as well as the difference between each point of input data and the polynomial values for the same abscissas. From considerations of mean differences, the fourth-degree curve was found to give the best fit of the four curves considered. For the case when Z is expressed as kilometers, we find

$$f(Z) = A + BZ + CZ^2 + DZ^3 + EZ^4 \qquad \text{(A-4)}$$

where f(Z) is expressed as m′

$$A = 0.4858124 \times 10^{-2}\ m'$$

$$B = 0.1338918 \times 10^{-3}\ m'/km$$

$$C = 0.1903029 \times 10^{-4}\ m'/km^2$$

$$D = 0.8288881 \times 10^{-9}\ m'/km^3$$

$$E = 0.1822113 \times 10^{-10} m'/km^4$$

When Z is expressed in meters, the values of the several coefficients are multiplied by $(10^{-3})^x\ (km/m)^x$, where x is the power of Z in the term with which the particular coefficient is associated. In both instances the dimensions of f(Z) is geopotential meters. A list of the functional values of f(Z) as defined above and a list of the difference between the graphical and functional value of f(Z) are also given in Table A-5.

It is noted that the function determined does not pass exactly through zero at Z = 0, but has a value of 0.004858124 meter at this altitude. This value, in effect, increases the value of the entire function by less than five thousandths of a meter, an amount which is trivial in the determination of geopotential to the nearest hundreth of a meter. This discrepancy undoubtedly results because the graphically derived values of f(Z) have only limited accuracy. In addition, only three graphical values of f(Z) resulted from the study for the region 0 to 90 km in which region the basic data consisted of only those 90 tabulated values of Z for integral multiples of one km′. There are, however, approximately 600 tabular entries of Z and $H_{62/R}$ between 0 and 90 km′, and more than three graphical values of f(Z) could possibly have been obtained for that region. A better fit at Z = 0 might have been determined if all the available data had been used. No great error is made, however, if zero is assigned to the coefficient of the $Z^0$ term.

TABLE A-5

GRAPHICALLY DETERMINED ESTIMATES OF $f_1(Z)$ COMPARED WITH POLYNOMIALLY DETERMINED VALUES OF $f(Z)$

| Z (km) | Graphical Estimates Of $f_1(Z)$,m | Percentage Degree of Certainty | Functional Value of $f(Z)$,m | Graphical Value Minus Functional Value,m |
|---|---|---|---|---|
| .000E-50 | .00000E-50 | 100.000 | .4858124E-02 | -.4858124E-02 |
| .440E+02 | .42000E-01 | 90.576 | .4280204E-01 | -.8020470E-03 |
| .720E+02 | .13000E+00 | 96.923 | .1243193E+00 | .5680610E-02 |
| .112E+03 | .34200E+00 | 98.830 | .3421639E+00 | -.1639800E-03 |
| .164E+03 | .86000E+00 | 97.680 | .8471755E+00 | .1282441E-01 |
| .203E+03 | .14500E+01 | 98.620 | .1424355E+01 | .2564490E-01 |
| .215E+03 | .16450E+01 | 98.480 | .1640593E+01 | .4407000E-02 |
| .225E+03 | .18210E+01 | 98.050 | .1835597E+01 | -.1459790E-01 |
| .245E+03 | .22550E+01 | 99.334 | .2267670E+01 | -.1267070E-01 |
| .258E+03 | .25700E+01 | 99.222 | .2579805E+01 | -.9805100E-02 |
| .266E+03 | .27750E+01 | 99.097 | .2784586E+01 | -.9586400E-02 |
| .27[illegible]E+03 | .29550E+01 | 98.815 | .2971894E+01 | -.1689450E-01 |
| .372E+03 | .65100E+01 | 99.538 | .6506622E+01 | .3377400E-02 |
| .45[illegible]E+03 | .11010E+02 | 99.728 | .1097247E+02 | .3752700E-01 |
| .486E+03 | .12915E+02 | 99.652 | .1293304E+02 | -.8047000E-02 |
| .492E+03 | .13360E+02 | 99.776 | .1334955E+02 | .1045000E-01 |
| .518E+03 | .15265E+02 | 99.771 | .1525077E+02 | .1422500E-01 |
| .530E+03 | .16175E+02 | 99.725 | .1618200E+02 | -.7009000E-02 |
| .570E+03 | .19510E+02 | 99.897 | .1953848E+02 | -.2848800E-01 |
| .612E+03 | .23450E+02 | 99.787 | .2349433E+02 | -.4433200E-01 |
| .642E+03 | .26650E+2 | 99.625 | .2660026E+02 | .4973800E-01 |
| .676E+03 | .30405E+02 | 99.852 | .3041133E+02 | -.6333000E-02 |

## A-3 APPLICATION OF f(Z) TO THE DETERMINATION OF GEOPOTENTIAL FOR Z = 90 km

The function f(Z) when used in Equation (A-2) provides the means for computing the value of $H_{62/8}$ for specified values of Z. An evaluation of Equation (A-4) for Z = 90 km shows f(Z) to be 0.201 m′ when the coefficient A is taken to be zero. The corresponding value of H from Equation (A-2) is found to be 88743.335 m′. This value should be a close approximation to that value of geopotential at which the defining temperature-altitude profiles of the U.S. Standard Atmosphere 1962 are switched from being linear in terms of geopotential, in the lower regime, to being linear in terms of geometric altitude in the upper regime. The calculation of the pressure for this transition altitude is considered under the discussion related to pressures at critical altitudes at 90 and below 90 km in the next section.

### Application of f(Z) Data to the Determination* of an Expression for Z versus H

The calculation of geometric altitude in terms of integral multiples of one geopotential kilometer for the required tables for the upper regime of the Standard Atmosphere may not be made by means of Equation (A-2), involving the functional expression for f(Z), without an undesirable iteration process. If Equation (A-2) is solved for Z without considering f(Z) explicitly in terms of Z, one obtains

$$A = \frac{r[H + f(Z)]}{r - [H + f(Z)]} \quad . \tag{A-5}$$

Since f(Z) represents a set of values associated with a specific set of geometric altitudes, it is apparent that this same set of values may become f(H) by being associated with a corresponding specific set of geopotentials, related to the geometric altitude through Equation (A-2). With this transformation of f(Z) to f(H), Equation (A-5) becomes

$$Z_{62/8} = \frac{r[H + f(H)]}{r - [H + f(H)]} \tag{A-6}$$

The function f(H) is found from the same basic graphical data employed in finding f(Z). In this case the value −0.342 m′, for example, previously associated with 112,000 geometric meters in Table A-5 is now associated with 110,060.488 geopotential meters, i.e.,

$$\frac{rZ}{r+Z} - 0.342 = 110060.83 - 0.342 = 110,060.488 \text{ m}' \quad .$$

Similar relationships apply for each of the other data points. These revised data points presented in Table A-6 yield a new polynomial fit,

$$f(H) = AA + ABH + ACH^2 + ADH^3 + AFH^4 \quad , \tag{A-7}$$

*The development of Equations (A-5) through (A-6) and the Table A-6 form the basis for the geopotential tables included in the United States Standard Atmosphere Supplements, 1966 (Ref. A-3).

## TABLE A-6

### GRAPHICALLY DETERMINED ESTIMATES OF $f_1(H)$ COMPARED WITH POLYNOMIALLY DETERMINED VALUES OF $f_1(H)$

| Geopotential km | Graphical Estimates of $f_1(H)$,m | Percentage Degree of Certainty | Functional Value of $f_1(H)$,m | Graphical Value Minus Functional Value,m |
|---|---|---|---|---|
| .0000000E-50 | .00000E-50 | 100.000 | .2579651E-02 | -.2579651E-02 |
| .4369749E+02 | .42000E-01 | 90.576 | .4386026E-01 | -.1860263E-02 |
| .7119349E+02 | .13000E+00 | 96.923 | .1262009E+00 | .3799100E-02 |
| .1100604E+03 | .34200E+00 | 98.830 | .3441282E+00 | -.2128220E-02 |
| .1598744E+03 | .86200E+00 | 97.680 | .8482695E+00 | .1373044E-01 |
| .1967164E+03 | .14500E+01 | 98.620 | .1424622E+01 | .2537790E-01 |
| .2079644E+03 | .16450E+01 | 98.480 | .1640626E+01 | .4373300E-02 |
| .2173064E+03 | .18210E+01 | 98.050 | .1835452E+01 | -.1445220E-01 |
| .2359054E+03 | .22550E+01 | 99.334 | .2267215E+01 | -.1221530E-01 |
| .2479344E+01 | .25700E+01 | 99.222 | .2579188E+01 | -.9188600E-02 |
| .2553134E+03 | .27750E+01 | 99.097 | .2783888E+01 | -.8888100E-02 |
| .2617554E+03 | .29550E+01 | 98.815 | .2971135E+01 | -.1613520E-01 |
| .3514274E+03 | .65100E+01 | 99.538 | .6505996E+01 | .4003100E-02 |
| .4254674E+03 | .11010E+02 | 99.728 | .1097266E+02 | .3733800E-01 |
| .4514694E+03 | .12925E+02 | 99.652 | .1293344E+02 | -.8443000E-02 |
| .4566424E+03 | .13360E+02 | 99.776 | .1334997E+02 | .1002700E-01 |
| .4789544E+03 | .15265E+02 | 99.771 | .1525125E+02 | .1374500E-01 |
| .4891954E+03 | .16175E+02 | 99.725 | .1618248E+02 | -.7483000E-02 |
| .5230754E+03 | .19510E+02 | 99.897 | .1953878E+02 | -.2878400E-01 |
| .5582304E+03 | .23450E+02 | 99.787 | .2349426E+02 | -.4426000E-01 |
| .5830824E+03 | .26650E+02 | 99.625 | .2659992E+02 | .5007600E-01 |
| .6109914E+03 | .30405E+02 | 99.852 | .3041092E+02 | -.5925000E-02 |

where f(H) is expressed as m'

$$AA = 0.2579651 \times 10^{-2} \ m'$$
$$AB = 0.2161710 \times 10^{-7} \ m'/m'$$
$$AC = 0.1807561 \times 10^{-10} \ m'/(m')^2$$
$$AD = 0.9153012 \times 10^{-16} \ m'/(m')^3$$
$$AE = 0.2006785 \times 10^{-22} \ m'/(m')^4$$

and where H is the altitude in geopotential meters. If H is to be expressed in kilometers, the values of the various coefficients must be multiplied by $(10^3)^x$ $(m'/km')^x$ where x is the power to which H is raised in that term to which the coefficient applies.

Equation (A-7) introduced into Equation (A-6) now yields values of Z for integral values of H in substantial agreement with the related values tabulated in the U.S. Standard Atmosphere 1962. The application of this equation to the expansion of the Standard Atmosphere above 90 km is considered in the main body of this report.

APPENDIX B

```
C                                       PROGRAM FOR COMPUTING 1962 STANDARD-ATMOSPHERE
C                                       VALUES OF PRESSURE, TEMPERATURE,TEMPERATURE
C                                       GRADIENT, AND DENSITY AS A FUNCTION OF INTEGRAL
C                                       MULTIPLES OF ONE THOUSAND GEOPOTENIAL METERS,
C                                       FROM 90,000 TO 120,000 GEOPOTENTIAL METERS, AND
C                                       AS A FUNCTION OF INTEGRAL MULTIPLES OF 5000
C                                       GEOPOTENTIAL FEET, FROM 295000 TO 390,000
C                                       GEOPOTENTIAL FEET. PRESSURE IS COMPUTED IN
C                                       MILLIBARS IN BOTH THE METRIC AND ENGLISH TABLES,
C                                       WHILE TEMPERATURE, TEMPERATURE GRADIENT, AND
C                                       DENISITY ARE COMPUTED IN TERMS OF DEGREES KELVIN,
C                                       DEGREES KELVIN PER METER, AND KILOGRAMS PER CUBIC
C                                       METER RESPECTIVELY IN THE METRIC TABLES, BUT IN
C                                       TERMS OF DEGREES RANKIN, DEGREES RANKIN PER FOOT,
C                                       AND POUNDS PER CUBIC FOOT RESPECTIVELY IN THE
C                                       ENGLISH TABLES.   PROGRAM ORIGINALLY WRITEN BY
C                                       MRS MELLO. REVIEWED AND COMMENTED BY R.A.  MINZNER
C                                       NOVEMBER 1966.
      DIMENSION G(7),A(7),TMR(4),FL(4),D(4),P(4)
C                                       THE QUOTIENT OF THE SEA-LEVEL VALUE OF THE
C                                       MOLECULAR WEIGHT DIVIDIED BY THE UNIVERSAL GAS
C                                       CONSTANT IS DESIGNATED FMOR.
   18 FMOR=3.483676E-03
C                                       THE QUANTITIES AA,AB,AC, AD AND AE ARE THE
C                                       COEFFICIENTS OF THE POLYNOMIAL DEFINING FX, THE
C                                       CORRECTION TERM IN THE EXPRESSION FOR CONVERTING
C                                       GEOPOTENTIAL TO GEOMETRIC ALTITUDE.
      AA=0.
      AB=-.2161710E-07
      AC=.1807561E-10
      AD=.9153012E-16
      AE=.2006785E-22
C                                       TMR(1) THROUGH TMR(4) ARE THE DEFINED VALUES OF THE
C                                       MOLECULAR SCALE TEMPERATURE (DEGREES KELVIN) AT
C                                       90,100,110 AND 120 GEOMETRIC KILOMETERS ALTITUDE.
      TMR(1)=180.65
      TMR(2)=210.65
      TMR(3)=260.65
      TMR(4)=360.65
C                                       D(1) THROUGH D(4) ARE THE PUBLISHED STANDARD-
C                                       ATMOSPHERE VALUES OF DENSITY (KG PER CUBIC METER)AT
C                                       90, 100,110 AND 120 KILOMETERS (THESE VALUES
C                                       APPEAR NOT TO BE USED IN THIS PROGRAM).
      D(1)=3.170E-06
      D(2)=4.974E-07
      D(3)=9.829E-08
      D(4)=2.436E-08
C                                       FL(1) THROUGH FL(4) ARE THE DEFINED VALUES OF THE
C                                       DERIVATIVE OF TM WITH RESPECT TO Z (DEGREES
C                                       KELVIN PER METER) FOR THE FOUR LAYERS WHOSE BASES
```

```
C                                 ARE 90, 100, 110 AND 120 KM.
      PL(1)=5.E-03
      PL(2)=5.E-03
      PL(3)=10.E-03
      PL(4)=20.E-03
C                                 P(1) THROUGH P(4) ARE THE MILLIBAR PRESSURES AT
C                                 ALTITUDES 90, 100, 110 AND 120 KILOMETERS.
      P(1)=1.6498E-03
      P(2)=3.0075E-04
      P(3)=7.3544E-05
      P(4)=2.5217E-05
C                                 G(1) THROUGH G(7) ARE THE COEFFICIENTS OF THE
C                                 LAMBERT EQUATION FOR EXPRESSING THE VARITION OF THE
C                                 ACCELERATION OF GRAVITY WITH GEOMETRIC HEIGHT Z
C                                 AT ABOUT 45 DEGREES LATITUDE.
      G(1)=9.80665
      G(2)=-3.0854195E-06
      G(3)=7.2539455E-13
      G(4)=-1.9167771E-19
      G(5)=2.9724620E-26
      G(6)=-5.5905936E-33
      G(7)=1.0219762E-39
      IF(SENSE SWITCH 1)19,20
C                                 HF IS THE HEIGHT IN GEOPOTENTIAL FEET OF THE
C                                 INITIAL ENTRY IN THE ENGLISH TABLES.
      HF=295000.
C                                 H=89916 IS THE HEIGHT IN GEOPOTENTIAL METERS
C                                 EQUAL TO 295,000 GEOPOTENTIAL FEET (EXACT).
   20 H=89916.
      GO TO 21
C                                 H=90.E+03 IS THE HEIGHT IN GEOPENTIAL METERS OF
C                                 THE INITIAL ENTRY IN THE METRIC TABLES.
   19 H=90.E+03
C                                 R IS THE EFFECTIVE EARTH RADIUS (FOR  PURPOSES OF
C                                 RELATING GEOMETRIC HEIGHT AND GEOPOTENTIAL) AT 45
C                                 DEGREES 32 MINUTES AND 33 SECONDS LATITUDE.
   21 R=6356.766E+03
      BEGIN TRACE
C                                 FX IS THE CORRECTION TERM IN THE H TO Z CONVERSION.
   11 FX=AA+AB*H+AC*H**2+AD*H**3+AE*H**4
C                                 THE NEXT EXPRESSION CONVERTS GEOPOTENTIAL H TO
C                                 GOEMETRIC HEIGHT Z.
      Z=R*(H+FX)/(R-(H+FX))
C                                 THE NEXT 14 STATEMENTS INVOLVE THE SETTING OF
C                                 INDICES FOR THE FOUR LAYERS AND FOUR REFERENCE
C                                 LEVELS OF THE CALCULATIONS.
      IF(Z-100.E+03)2,3,4
    2 K=1
      ZR=90.E+03
      GO TO 5
```

```
    3 K=2
      ZR=100.E+03
      GO TO 5
    4 IF(Z-110.E+03)3,6,7
    6 K=3
      ZR=110.E+03
      GO TO 5
    7 IF(Z-120.E+03) 6,28,28
   28 K=4
      ZR=120.E+03
C                                 THE NEXT 23 STATEMENTS DEAL WITH THE CALCULATION OF
C                                 PRESS,THE PRESSURE IN MILLIBARS, IN ACCORDANCE
C                                 WITH THE CORRECTED VERSION OF EQUATION 27 OF
C                                 CHAMPION AND MINZNER, REV. OF GEOPHYSICS 1,P57,1963
    5 SUMA0=0.
      SUMA1=0.
      SUM=0.
      DO 8 I=1,7
      A(I)=(TMR(K)/FL(K)-ZR)**(I-1)
    8 SUMA0=SUMA0+(-1.)**(I-1)*G(I)*A(I)
      DO 9 L=1,6
      GL=L
      DO 10 I=L,6
      M=I+L
      N=I-L+1
   10 SUM=SUM+(-1.)**(M)*G(I+1)*A(N)
      SHEL=Z**L-ZR**L
      SHEL=SHEL/GL*SUM
      SUMA1=SUMA1+SHEL
    9 SUM=0.
      SHE=-SUMA0*(FMOR/FL(K))
      SHEIL=SUMA1*FMOR/FL(K)
      PRESS=TMR(K)+FL(K)*(Z-ZR)
      PRESS=PRESS/TMR(K)
      PRESS=PRESS**SHE
      PRESS=PRESS*P(K)
      PRESS=PRESS*EXP(-SHEIL)
C                                 THE NEXT STATEMENT EXPRESSES THE MOLECULAR SCALE
C                                 TEMPERATURE TM AT ALTITUDE Z WITHIN ANY LAYER K.
      TM=TMR(K)+FL(K)*(Z-ZR)
C                                 THE NEXT STATEMENT EXPRESSES THE DENSITY IN KG. PER
C                                 CUBIC METER IN TERMS OF TM AND PRESS USING THE
C                                 GAS LAW. THE FACTOR 1.E+02 IS REQUIRED TO CONVERT
C                                 MILLIBAR PRESSURES TO NEWTONS PER SQUARE METER, THE
C                                 PROPER UNITS OF PRESSURE IN THE MKS SYSTEM OF UNITS
      DENS=PRESS/TM*FMOR*1.E+02
      IF(SENSE SWITCH 1 )22,23
C                                 THE NEXT STATEMENT CONVERTS TM IN DEGREES K TO TMD
C                                 IN DEGREES RANKIN.
   23 TMD=1.8*TM
```

```
C                                  THE NEXT STATEMENT CONVERTS DENSITY IN KILOGRAMS
C                                  PER CUBIC METER TO POUNDS PER CUBIC FOOT.
      DENS =DENS*6.242797E-02
C                                  THE NEXT STATEMENT ROUNDS THE QUANTITY TO TWO
C                                  DECIMAL PLACES.
      TMD=TMD+.005
C                                  THE NEXT STATEMENT CONVERTS Z IN METERS TO ZF IN-
C                                  FEET.
      ZF=Z*3.2808399
C                                  THE NEXT STATEMENT ROUNDS THE QUANTITY TO TWO
C                                  DECIMAL PLACES.
      ZF=ZF+.005
C                                  THE NEXT STATEMENT CONVERTS TM IN DEGREES KELVIN TO
C                                  TMC IN DEGREES CELCIUS.
      TMC=TM-273.15
C                                  THE NEXT STATEMENT ROUNDS THE QUANTITY TO TWO
C                                  DECIMAL PLACES.
      TMC=TMC+.005
C                                  THE NEXT STATEMENT CONVERTS FL IN DEGREES KELVIN
C                                  PER METER TO RL IN DEGREES RANKIN PER FOOT.
      RL=FL(K)*.54864
C                                  THE NEXT STATEMENT IS THE PUNCH STATEMENT FOR THE
C                                  ENGLISH TABLES.
      PUNCH 101,HF,ZF,RL,TMD,TMC,PRESS,DENS
      GO TO 24
C                                  THE NEXT TWO STATEMENTS ROUND THE RESPECTIVE
C                                  QUANTITIES TO TWO DECIMAL PLACES, WHERE FZ IS THE
C                                  ROUNDED VALUE OF Z IN METRIC UNITS.
   22 TM=TM+.005
      FZ=Z+.005
C                                  THE NEXT STATEMENT IS THE PUNCH STATEMENT FOR THE
C                                  METRIC TABLES.
      PUNCH 100,H,FZ,FL(K),TM,PRESS,DENS
C                                  THE NEXT 8 STATEMENTS SET THE INCREMENTS OF BOTH
C                                  THE METRIC AND ENGLISH TABLES.
   24 IF(SENSE SWITCH 1)12,13
   12 IF(H-120.E+03)14,15,15
   15 PAUSE
      GO TO 18
   14 H=H+1.E+03
      GO TO 11
   13 IF(Z-120.E+03)16,17,17
   16 HF=HF+5.E+03
C                                  THE NEXT STATEMENT CONVERTS HF GEOPOTENTIAL HEIGHT
C                                  IN FEET TO GEOPOTENTIAL HEIGHT IN METERS PRIOR TO
C                                  CALCULATING THE THERMODYNAMIC PROPERTIES FOR HF.
      H=.3048*HF
      GO TO 11
  100 FORMAT(2F11.2,F6.3,F8.2,E12.5,E11.4)
  101 FORMAT(2F11.2,F10.7,2F8.2,E12.5,E11.4)
      END TRACE
      END
```

## REFERENCES

1. United States Standard Atmosphere, National Aeronautics and Space Administration, U.S. Air Force, and U.S. Weather Bureau, Government Printing Office, Washington, D.C. (1962).

2. Champion, K.S.W., Minzner, R.A., "Revision of United States Standard Atmosphere 90 to 700 km," Rev. of Geophys. 1, 57-84 (1963), also AFCRL-62-802 (1962).

3. Minzner, R.A., Ripley, W.S., "The ARDC Model Atmosphere 1956," Air Force Surveys in Geophysics No. 86, AFCRC TN-56-204, ASTIA Document 110233 (1956).

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13. ABSTRACT

The "United States Standard Atmosphere, 1962," was published with two kinds of metric-unit tables for the altitude interval from -5000 to 90,000 meters. One kind of table presented the atmospheric properties as a function of integral multiples of particular numbers of geopotential meters while the second presented the atmospheric properties as a function of integral multiples of semilar numbers of geometric meters. For the region above 90,000 meters, altitude only one type of metric table was published. This type presented atmospheric properties in integral multiples of particular numbers of geometric meters. meters. A similar situation prevailed for the English-unit tables. The need for both metric-unit and English-unit tables as a function of integral multiples of specific numbers of geopotential meters for altitudes above 90 kilometers has prompted a new set of calculations, which required the use of equations not specifically presented in the United States Standard Atmosphere, 1962. The development of these equations is discussed and the value of all constants employed are given. The calculations involve a transformation between geopotential and geometric altitude, and the development of an empirical analytical expression relating these quantities is presented. This empirical function yields results which differ by less than 0.1 meter at 700 km altitude, from those computed in an unspecified manner for the United States Standard Atmosphere, 1962.

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